

# Facial nonrepetitive colourings of plane graphs

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A sequence  $a_1, a_2, \dots, a_{2n}$  such that  $a_i = a_{n+i}$  for all  $1 \leq i \leq n$  is called *repetition*. A sequence  $S$  is called *nonrepetitive* if no its subsequence of consecutive terms of  $S$  is a repetition. Let  $G$  be a graph whose vertices (edges) are coloured. A simple path (a trail) in  $G$  is called vertex (edge) nonrepetitive if the sequence of colours of its vertices (edges) is non repetitive (see [1]). Let  $G$  be a connected plane graph. Then the *facial nonrepetitive vertex (edge) colouring* of  $G$  is a vertex (an edge) colouring such that any facial path (trail), i.e. a path (a trail) of consecutive vertices (edges) on the boundary walk of any face of  $G$ , is vertex (edge) nonrepetitive. Let  $\pi'_f(G)$  be the minimum number of colours of a facial nonrepetitive edge colouring of a plane graph  $G$ . We show (see [3]) that for any bridgeless connected plane graph  $G$  there exists an absolute constant  $K$  such that

$$\pi'_f(G) \leq K \leq 8.$$

Analogously one can define  $\pi_f(G)$  to be the minimum number of colours in a facial vertex nonrepetitive colouring of a connected plane graph  $G$ , see [2].

We conjecture that also in this case there is an absolute constant  $C$  such that  $\pi_f(G) \leq C$ .

We show that this conjecture is true for Halin graphs, plane hamiltonian graphs, (and hence also for) 4-connected plane graphs, and for 2-connected cubic plane graphs.

## References

- [1] N.Alon, J.Grytczuk, M.Haluszczenk, O.Riordan, *Non-repetitive colorings of graphs*, Random Structures Algorithms 21 (2002) pp.336-346.
- [2] J.Harant, S.Jendroľ, *Facial non-repetitive vertex colourings of some families of 2-connected plane graphs*, (submitted).
- [3] F.Havet, S.Jendroľ, R.Soták, E.Škrabul'áková, *Facial non-repetitive edge-colouring of plane graphs*, J. Graph Theory (to appear).