

List Coloring of Simple Hypergraphs

Alexandr Kostochka

The *list chromatic number* $\chi_\ell(G)$ of a hypergraph $G = (V, E)$ is the minimum integer s such that for every assignment of a list L_v of s colors to each vertex v of G , there is a vertex coloring of G in which the color of each vertex is in its list and there are no monochromatic edges. Before 2000, Alon proved that every graph with “high” average degree cannot have small list chromatic number. We prove that the same holds for simple r -uniform hypergraphs with a different notion of “high”. Recall that a hypergraph is simple if no two edges have more than one common vertex. Note that non-simple n -vertex r -uniform hypergraphs may have average degree about $(n/r)^{r-2}$ and still be 2-list-colorable. We use the result to prove that for any finite set of points X in the plane, and for any finite integer s , one can assign a list of s distinct colors to each point of the plane, so that any coloring of the plane that colors each point by a color from its list contains a monochromatic isometric copy of X . This joint work with Noga Alon.