Closures for paths and cycles in graphs

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The closure concept for hamiltonicity in claw-free graphs has proved to be a useful tool for studying cycle and path properties of claw-free graphs. However, its applicability is restricted only to properties $\mathcal{P}$ that are stable under the closure operation, i.e. such that a graph G has $\mathcal{P}$ if and only if the closure of G has $\mathcal{P}$. A well-known example of an ustable property is Hamilton-connectedness.

In the talk we first survey recent applications of the closure for hamiltonicity to various cycle problems (such as degree conditions or connectivity bounds and the Thomassen/Matthews-Sumner conjectures) and recent development in the field. Then we concentrate on variations of the closure that are applicable to some other cycle and path properties (Hamilton-connectedness, 2-factor). As an application, we e.g. show that every 7-connected claw-free graph is Hamilton-connected, we obtain asymptotically sharp degree conditions for Hamilton-connectedness and we present recent progress in forbidden pairs for Hamilton-connectedness and discuss remaining open cases.