Decompostion of graphs into forests

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The fractional arboricity of a graph G is defined as $\gamma_f(G) = \max|E(G[X])|/(|X|-1)$, where the maximum is over all the subsets X of $V(G)$. The well-known Nash-William Theorem says that for $k = \lceil \gamma_f(G) \rceil$, G decomposes into k forests, i.e., $E(G)$ is the union of the edge sets of k forests. The result is sharp in the sense that G does not decompose into $k-1$ forests. On the other hand, by taking the ceiling of the fractional arboricity, some information about the graph contained in the parameter is lost. In this talk, we propose a conjecture in the attempt to use some extra information contained in the fractional arboricity. The conjecture says that if $\gamma_f(G) \leq k+d/(k+d+1)$, then G decomposes into $k+1$ forests with one of them having maximum degree at most d. The conjecture is sharp in the sense that for any $\epsilon > 0$, there is a graph G with $\gamma_f(G) \leq k+d/(k+d+1)+\epsilon$ and G has no such decomposition. The conjecture is verified for the cases $k = 1$ and $d \leq 6$.